

Harmonious Proportion and Ratio

Since therefore, all things are beautiful and to some measure pleasing; and [since] there is no beauty and pleasure without proportion, and proportion is to be found primarily in numbers; all things must have numerical proportion. Consequently, number is the principal exemplar in the mind of the Creator and as such it is the principal trace that, in things, leads to wisdom. Since this trace is extremely clear to all and is closest to God, it leads us to Him through the seven differences¹ and it causes us to know Him in all corporeal and sensible things; and while we learn that things have numerical proportion, we take pleasure in this numerical proportion and we judge things irrefutably by virtue of the laws that govern it.

St Bonaventure, *Itinerarium mentis in Deum*, II, 7

The perception of beauty is based in part upon the recognition of harmonious relationships that are consistent with the divine order – ‘due proportion’. Just as described in regard to the application of number, for proportion to be *due* proportion, the symbolism must be in accordance with the end to which the object is directed.

The expansion of numbers into geometric form is one way of expressing the beauty of number as applied to harmonious relationship. For example, four into a square, immediately creates an ordered relationship between four lines of equal magnitude. Relationship can also be described numerically – the beauty of the relationship is derived from the magnitude of one property relative to another. For example, if we take a room of relative length, breadth and height, 3, 2 and 1 respectively, the relationship between these values is consistent with traditional ideas of harmony. Provided that the proportions of each relative to the other two do not change, then it allows for infinite variation in the size of this building.

Proportion and ratio employed in accordance with traditional ideas of ‘due proportion’ can be employed in any work of man that potentially can be imbued with beauty: art, architecture, music, literature, gardening, in fact any works of man, no matter how mundane.

The difference between numerical ratio and numerical proportion

The tradition distinguishes between ratio and proportion. We shall use the terminology of Boethius’ book *De Institutione Arithmetica*² which was an influential source on number theory up to and including the Renaissance³.

¹ In his *Itinerarium*, Bonaventure classifies numbers in seven categories – ‘differences’ – that stimulate us in different ways, all playing their part in our journey to God.

² *Boethian Number Theory – A Translation of the De Institutione Arithmetica*, by Michael Masi, Rodopi, NY 2006

³ cf *Boethius: The Consolations of Music, Logic, Theology and Philosophy*, Henry Chadwick, Clarendon 1990; *The Gothic Cathedral*, Otto von Simson, Harper and Row, New York 1964; as well as the introduction to Michael Masi’s translation of *De Institutione Arithmetica*, New York, 2006

If you have two numbers in relation to each other they constitute a *ratio*. This would be expressed mathematically, for example 1:2. Such a relationship can possess a beauty in itself: strings of relative length 1 and 2 respectively will produce notes that combine to produce a musical interval called a perfect fifth.

Proportionality is something distinct from ratio. Boethius defines proportionality as a similar relationship of two or more ratios. For example, the ratio 2:3, is said to be in proportion to 3:4 because the numerical difference between the first and second number in each ratio is equal to 1 (that is $3-2 = 4-3$). This is referred to as an ‘arithmetic’ proportion. Harmonious proportions are usually expressed using three numbers only where the largest number of the first ratio, in this case 3, is the same as the smaller number of the second ratio. So rather than presenting the particular arithmetic harmony as 1:2 and 2:3, it is written as 1:2:3. The middle term, in this case 2, is described as the ‘arithmetic mean’ of 1 and 3. Boethius describes another 9 types of harmonious proportion in addition to the arithmetic proportion. All 10 will be discussed in more detail later.

All these proportions are to be viewed as identifiable aspects of the divine Proportion that rests in the Trinity, is dispersed throughout creation and which have been discovered, rather than developed, by man.

The sources of ratios and proportionality that is in accord with the divine order

I am aware of four primary sources for knowledge of proportion and ratio:

1. Arithmetic
2. The Bible
3. Geometry
4. Music for which there are three sub-categories: celestial, human, and instrumental. These will be explained later.

There is a deal of overlap and the proportions that appear most commonly, in architecture for example, are those that can be generated from different sources independently, thereby increasing the likelihood, in the minds of the architect, that they are reproducing the divine order.

1. Arithmetic

Table 1, at the end of the article, contains the 10 harmonious proportions of Boethius. They are ordered in five pairs. In each pair, the first established proportion is referred to as a ‘leader’ or ‘major’ proportion. The second proportion in each pair, derived from the first (for example by the inversion of one of the ratios in the major) is termed a ‘follower’ or ‘minor’ proportion; sometimes Boethius refers to these as being ‘contrary’ to a major.

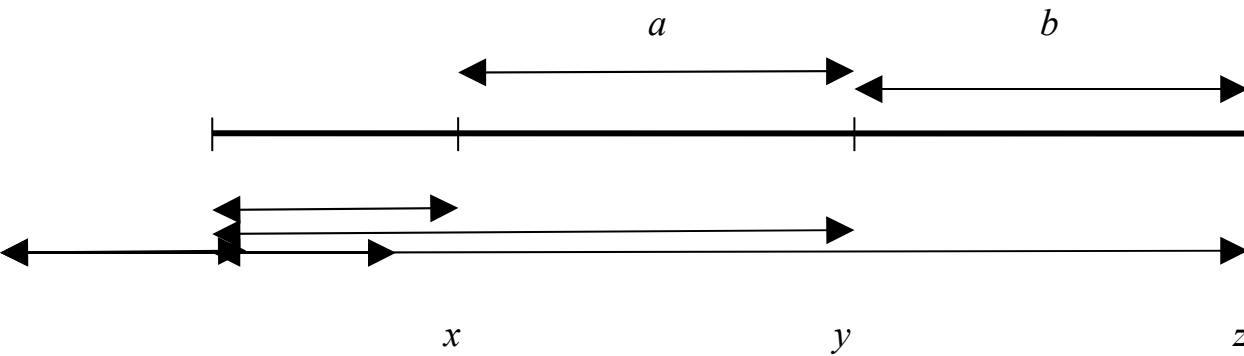
Boethius not only saw significance in the nature of each proportion, but also in the fact that there are 10 of them.

‘It is testified to and known among the ancients who have studied the learning of Pythagoras, or Plato, or Aristotle that there are three ways to knowledge: arithmetic, geometric, harmonic⁴. After these relationships of proportions there are three others, which are conveyed to us without names but are called fourth, fifth and sixth and which are contrasted with the above. Then later thinkers, on account of the perfection of the number 10, which was pleasing to the Pythagoreans, added four other kinds, so that in these proportionalities they brought together the body of proportions 10 in number.’⁵

As already mentioned, to describe proportionality a minimum of three terms are required. There is no maximum number of terms – in principle you could have an infinite series corresponding to, for example, geometric proportion. Boethius described each proportion in prose, rather than with mathematical symbols. So, for example, arithmetic proportionality is described as follows:

‘We call that an arithmetic medial proportion which as often as in three or any number of stated terms there is found the same and equal difference between all the terms.’

The words of Boethius have been converted into mathematical formulae here, as they will, it is presumed, be more easily understood by the modern reader. As three is the minimum number terms needed to define the proportionality, we will define all the proportion using these three terms x , y and z . We will call the interval between x and y , that is $y - x$, a ; and the interval between y and z , that is $z - y$, b . This is represented diagrammatically as follows.



The 10 proportional relationships of Boethius are presented in table 1. As Boethius has explained above, some are more important than others. The first three (arithmetic, harmonic and geometric) are considered important because they can all be constructed out of the first four numbers, 1, 2, 3, and 4, which also combine to make the tetractys. From these four numbers, combined in the 3

⁴ The word ‘harmonic’ refers to a particular proportion and should not be confused with the generic term for due proportion – ‘harmonious’ proportion.

⁵ De Institutione Arithmetica, Bk2, Ch41

proportionalities, come the main ratios from which the basic chords and intervals of instrumental music can be derived. Also, they appear in the dimensions of the Temple of Solomon.

The importance of the final proportionality, the Fourth of Four, is demonstrated by its appearance in the form 6 : 10 : 16, corresponding to the proportions of the idealized human body, and the proportions of the ark made by Noah in the Book of Genesis in the bible and interpreted by St Augustine as corresponding to the proportions of the figure of Christ. It is noteworthy that this series is an earlier form of what was presented much later by the Italian mathematician Fibonacci in his famous series. As with the Fibonacci series, as the Fourth of Four progresses *ad infinitum*, the ratio of adjacent terms tends towards the ratio known today as the Golden Section⁶.

Another seminal text used as an authority on harmonic ratios and proportionalities is contained in a 1st century Roman textbook on architecture, Vitruvius's *10 Books on Architecture*⁷. This describes the proportionality based upon musical harmonies and the human ideal proportions as a basis for the architect. It is an important text because it gives us confirmation that some of the harmonies described by the Greek philosophers (and subsequently by Boethius) were employed by Roman architects. The book itself has not always been a primary source available to the Christian world, but it does give a strong indication as to which proportional relationships were used by the ancient world, and so these proportions would have been widely known and available for adoption by the Christian world and handed on after that without reference to Vitruvius. This book was rediscovered in the 15th century and was used as a source book for architects for centuries. For example, it was the inspiration for Andrea Palladio, the 16th century Italian architect, from which he developed his 'palladian' style. Because Vitruvius's book was not Catholic, non-Catholics in for example post-Reformation England were happy to use his methods because they contained the secrets to classical beauty in architecture. It was translated into English in the 17th century under the name *Vitruvius Britannicus*. This became the basis for harmonious design in British architecture that shaped the English architecture from this period through the early 20th century. English Georgian architecture, for example, is derived from it⁸.

Geometry

The fundamental geometric representation of significant numbers, such as a square or a triangle, can be geometrically expanded and subdivided to form harmonic relationships. By constructing a grid that consists of a latticework of equilateral triangles, for example, one can selectively choose the points on the grid as key points in any design – for example, the corners of a room, or the point at which the wall joins the roof – confident that the building or picture will be consistent with the divine order. This generation of design through the multiplication of a basic geometric

⁶ This is discussed in greater detail in another article, which will follow this.

⁷ An English translation is published by Dover Press, 1960

⁸ For more detail see Robert Tavernor, *Palladio and Palladianism*, Thames and Hudson, London, 1991

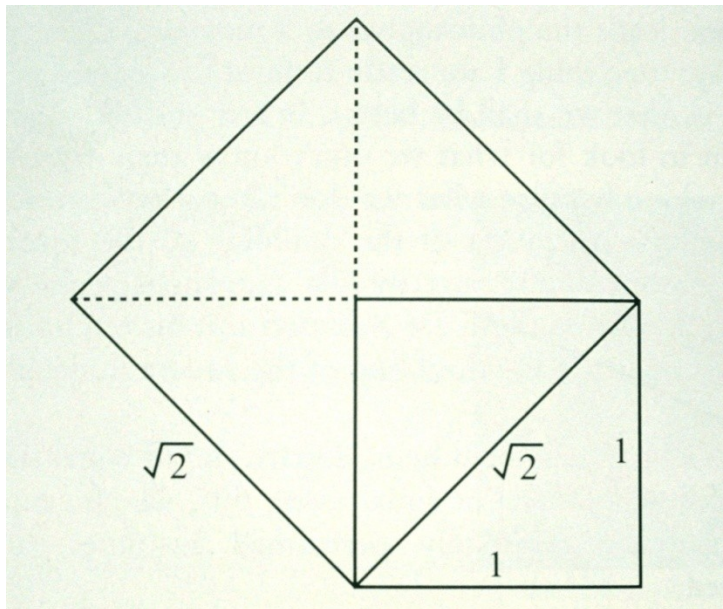
shape is the basis for geometric patterned art found in Christian culture, for example the Westminster pavement or in San Miniato al Monte in Florence.

Documents from medieval masons, the 13th century Frenchman, Villard de Honnecourt, the German Matthew Roriczer and a letter describing the design of Milan cathedral (both dating from the 15th century) describe these geometric processes as well.

Quadrature and moral significance of proportionality

One example of the geometric generation of form is the process known as ‘quadrature’. Essentially a square twice the area of another is created by constructing a second, based upon the diagonal of the first. Conversely, one can construct a square half the area of the first by joining the mid-points of sides of the first. In each case the ratio of the length of the sides of the larger to the smaller is $1:\sqrt{2}$.

The quadrature method for harmonious design, using only the traditional tools of set-square and compass, persisted throughout the medieval period. Sometimes the ratio was approximated to 17:12.



$\sqrt{2}$ is an irrational number. The Pythagorean tradition of numbers dislikes ‘irrational’ numbers – those that cannot be expressed numerically using whole integers. However, the idea, at least, of $\sqrt{2}$ can be conceived as a precise relationship through geometry, in the form of a ratio of the lengths of two lines, as constructed left. The medieval cathedral builders were familiar with this ratio which they constructed geometrically and employed in the design of spaces and arches to evoke a sense of harmony in the building.

They saw $\sqrt{2}$ as representing the temperate mean between two extremes, 1 and 2, because the ratios $1:\sqrt{2}$ and $\sqrt{2}:2$ are numerically equal. That is $1:\sqrt{2}:2$ constitutes a geometric proportion, as described by Boethius.

In this construction, the two squares defining the outer and inner dimension of the cloistered walkway around a quadrangle were built in accordance with this dimension which meant that the walkway had the same area as the inner square. So, symbolically, walking through the cloister to the church, could be seen as treading the temperate path of virtue to heaven, between two sinful

extremes. The rate of reduction of the width of the spire conforms to the same symbolic order – pointing upwards to heaven on the line of the temperate mean.

In fact, any of the beautiful proportions that are expressed arithmetically can be interpreted as a representation of moral beauty. In each case the mean corresponds to the temperate mean between two extremes. The symbolism is all the more poignant in the case of quadrature, however, because the two extremes 1 and 2 represent together separation and disharmony. They are harmonized by the geometric construction of their mean. The beauty of the geometric patterns, cloisters and spires produced using this proportionality could be interpreted as a visual demonstration of the truth that ‘there is nothing to prevent human nature’s being raised up to something greater, even after sin. God permits evil in order to draw forth some greater good. Thus St Paul says, “Where sin increased, grace abounded all the more”; and the *Exultet* sings, “Oh happy fault,... that gained for us so great a Redeemer!”’⁹

Music

The general principle of the musical law as the source of all beauty comes into the Christian tradition in detailed form from St Augustine¹⁰. Another influential and much later figure in this regard is the 9th century neo-Platonist John Scotus Eriugena. Eriugena’s commentary on the Celestial Hierarchy of Dionysius the Pseudo-Areopogite was influential on Abbot Suger of St Denis near Paris who built the first gothic church in the 12th century and who pioneered the gothic style that dominated for the next 300 years. He describes the laws of harmonic proportion by which the different parts of the universe are reconciled. Suger placed this idea at the head of his treatise about the building and consecration of his church¹¹.

In Boethius’s first (and only extant) book on music he considers how the senses are fundamental to humanity:

‘Perception through the senses is so spontaneously and naturally present in certain living things that it is impossible to think of an animal apart from sense.’¹²

He observes that music influences the mood and therefore influences the morality of the listener. He then remarks:

‘Thus we conclude too that not without reason did Plato say that the soul of the universe was composed according to musical harmony. For since, in accord with what is united and adjusted harmoniously in us we take out that which is joined together in

⁹ St Thomas Aquinas, *Summa Theologiae* III, 1, 3 ad 3; cf Rom 5:20; from Catechism of Catholic Church, 412

¹⁰ In his book *De Musica*

¹¹ Described by Otto von Simson, *The Gothic Cathedral*, p 126, Harper and Row, 1964

¹² Boethius: *Five Books on Music*, Bk 1, Ch1

sounds suitably and harmoniously and delight in that, we recognize that we ourselves have been put together with the same likeness. For similarity is pleasing, but dissimilarity is hateful and opposed.’

For Boethius, as Augustine before him, we are made by God to perceive the divine order. When we do so we apprehend beauty. The fact that the beauty of certain musical intervals can be related to a harmonious order indicates that numerical order pervades all that is beautiful including man himself. Our ability to perceive this numerical order exists because we are constructed similarly. The ‘structure’ of the human person refers not only to his physical body, but his soul and the relationship between them. The human order is compatible with the audible musical order through their common participation in the divine order. He then extends his point, arguing that we perceive beauty in the cosmos because it participates in this divine harmony too. He calls each of these manifestations of the divine order ‘music’.

‘There are three types of music in which the power of music is manifested. The first is cosmic; the second is human, the third is that which is produced by means of instruments [including the human voice] as the cithara of the tibia and the others of this nature which are the servants of song.

‘And first of all, that which we call cosmic is to be contemplated mainly in the celestial phenomena, or in the structure of the elements, or in the sequence of the seasons.

..

‘But whoever turns his attention to his own self, understands human music. For what is it that unites that incorporeal vital force of the reason with the body, unless it be a certain accurate joining together, and as it were, proportion, low and high tones, so producing one consonance?’¹³

In this passage, there is an assumption that there is a sound that is produced by the motions of planets though inaudible to us here on earth. We would not subscribe to this today, but the general idea of music referring to the form of the divine order in both mankind and the sensible world is just as applicable today as it always was.

Instrumental musical harmony is treated both in his book on music and his book on arithmetic, when he relates them the arithmetic, geometric and harmonic proportions. Each number represents the relative length of a string that when plucked produced a note, or alternatively, the relative positions of the holes that can be stopped with the finger in a wind instrument.¹⁴ There is an

¹³ Boethius: *Five Books on Music*, Bk 1, Ch2

¹⁴ See also, Plato’s *Timeaus*, who in turn attributes them to Pythagoras.

assumption underlying this that there is a broad consensus that certain fundamental consonances are perceived as beautiful.

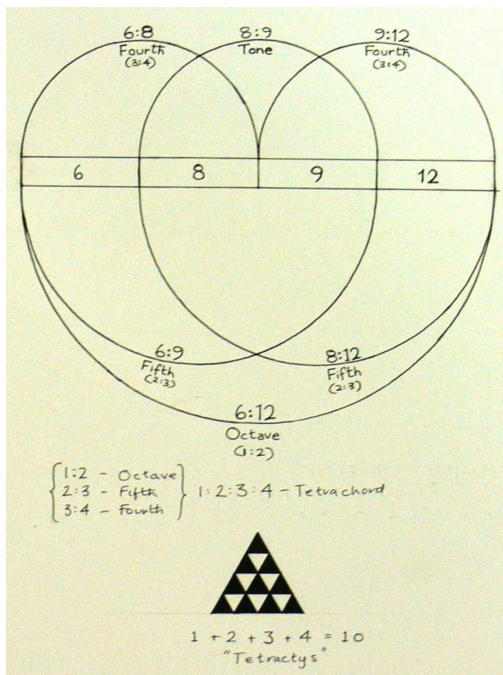
All but the interval corresponding to a single tone are generated from the number 10, presented as a tetractys : 1, 2, 3, 4. Each number in ratio to its neighbour produces a fundamental consonance – one of the basic harmonious chords or intervals upon which instrumental music was built. If we imagine four strings of these relative lengths we can construct the purest harmonies by plucking any two strings together. The fundamental consonances are the octave, 1:2; perfect fifth, 2:3; and perfect fourth, 3:4. There are secondary harmonic ratios, 1:3 and 1:4, which are called ‘triple’ and ‘quadruple’ that can be made by the combination of an octave and a perfect fifth ($1:2 \times 2:3 = 2:6 = 1:3$) and two octaves $1:2^2 = 1:4$, respectively. The ratio 8:9 is significant in that it constitutes an interval of a single tone: ie multiplying a string by the ratio 8:9 raises the pitch by a tone.

The grouping of these basic harmonies is called a tetrachord. The term *tetrachord* literally means *four strings*.

Incorporating the musical ratios into proportionality

Because the tone, in the ratio 8:9, cannot be expressed with the basic numbers of the tetractys (1, 2, 3, 4) another arithmetic presentation was developed. First, the octave ratio, 1:2 was multiplied by 6 to give the ratio 6:12. Then the arithmetic and geometric means of 6 and 12 are calculated: 9 and 8 respectively. This gives us a series of four numbers 6, 8, 9, 12. The ratios generated by relating each number with the other three produces the fundamental Pythagorean harmonies: octave (6:12), perfect fourth (9:12), perfect fifth (8:12) plus the tonal interval (8:9). This is often represented in a diagram left, which appears in Boethius’s book and, several hundred years later held by Pythagoras in a detail of Raphael’s fresco in the Vatican, the School of Athens.

Each of these ratios is found in Vitruvius’s textbook on architecture¹⁵, who also mentions the additional ratio 3:8, derived



¹⁵ Book 5, Ch 4



Details from Raphael's School of Athens

from an octave combined with a fourth, which to my knowledge is not mentioned by Boethius; and the proportionality called Fourth of Four in Boethius's book on arithmetic ($6 : 10 : 16$). The 13th century gothic architect, Villard de Honnecourt who worked for the Cistercian order included also the musical third, the ratio $4:5$ ¹⁶ and this taken by commentators such as von Simson to be a regularly used ratio in the middle ages. The ratio of $5:3$ (which also occurs as a fundamental ratio in 'human' music) forms what many consider to be the most beautiful interval, the major sixth.

Human music – $6 : 10 : 16$, the ideal human proportionality

As we have mentioned, Boethius felt that study of the human person would reveal the cosmic order, as much as study of the cosmos.

Although derived from ideas about the physical proportions of the body, this is only part of what proportionality of the 'human music' is trying to describe. 'Human music' refers to a divine resonance or harmony that is present when relating each aspect of the human person to the other, physical and spiritual. It also reflects a participation of human nature in the divine order, and points, ultimately, to our destiny of partaking of the divine nature in heaven. The most commonly used source for the classical numerical proportions is Vitruvius, which coincides with that of the final one in the list of Boethius's, called Fourth of Four, which is $6:10:16$.

Of the Church Fathers, Augustine particularly describes the Christian understanding of this 'human music'. He relates the human proportions to those of the ark built by Noah, as directed by God, and which is based upon the proportions of the ideal man, Christ. Six and 10 are seen as the perfect numbers that permeate the description of the human body.

¹⁶ Otto von Simson, *The Gothic Cathedral*, Harper and Row, 1964

‘[The ark] is certainly a figure of the City of God sojourning in this world; that is to say, of the Church, which is rescued by the wood on which hung the Mediator of God and men, Christ Jesus. For even in its very dimensions in length, breadth and height it represents the human body in which He came, as it had been foretold. For the length of the human body, from the crown of the head to the sole of the feet is six times its breadth from side to side, and ten times its depth or thickness, measuring from back to front: that is to say, if you measure a man as he lies on his back or his face, he is six times as long from head to foot as he is broad from side to side, and ten times as long as he is high from the ground. And therefore the ark was made 300 cubits long, 50 in breadth and 30 in height.’¹⁷

The Vitruvian ideal proportions are based upon the traditional measure of finger (digitus), palm (palms), foot (pes), and cubit (cubitus) – the length of the forearm. There are 10 fingers, 4 fingers make a palm, 4 palms make a foot and hence the foot contains 16 fingers. The importance of 16 is confirmed for Vitruvius in his observation that 6 and 10, each themselves perfect numbers, combine to make the most perfect 16¹⁸.

A survey of Christian art will quickly reveal that different traditions use a variety of proportions in the artistic representation of the human form. Even if a figure is sketched out by an artist in rough accordance with a canon of proportion, unless the posture is a very stiff pose with all limbs in the plane of the painting (as with ancient Egyptian art), the problem of having limbs at different angles and different degrees of foreshortening mean that the artist will rely strongly on their sense of what looks right as much as anything. If it is possible to generalise, the body proportions follow a general scheme that is used even by modern illustrators. However there are exceptions in the detail: iconographic styles tend to have a smaller head in proportion to the rest of the body length (around 1:8 or even smaller), while in more naturalistic styles it is larger (around 1:7). The facial features of the icon and the size of the hands are modified slightly also relative to the natural proportions.

It strikes me that the recognition in these dimensions of their arithmetic and biblical significance relates the harmony of man to the whole person: body, soul and heart¹⁹. The heart is the symbol of the core of us. The musical analogy is the creation of a chord when two notes are in audible contact with each other. Each note retains its integrity, but the chord is something created by their combination that is a distinct third entity. It is created out of nothing, so to speak. God is Love, and loving relationships, because they incorporate God, are productive and always generate something out of nothing. The heart is the resonating chord of humanity created by the harmony of body and soul, the vector sum, so to speak, of our thoughts and actions; our human centre of

¹⁷ St Augustine, *City of God*, Chapter 26 ‘That the ark that Noah was ordered to make figures in every respect Christ and His Church’

¹⁸ Vitruvius, *10 Books on Architecture*, Bk III, Ch 1

¹⁹ Cf. Catechism of the Catholic: 367, 368

gravity. It is the symbol of love, because to love is our divinely created purpose. The more that the body and soul of a person are in harmony with the divine order, and so in harmony in each other; the more the heart they combine to form resembles that Sacred Heart of the ideal man, Christ in his humanity. This is another reason why harmonious proportion is always expressed as three numbers. Three entities are required to reflect divine love and so the numerical relationship that is patterned on the divine order must have three numbers too. So a beautiful combination of numbers – a ratio that reflects the divine order – will generate a third number and create a harmonic proportion.

It is the Trinity that is the perfect form for loving relationship. The Love that exists between two persons is itself a person of the Trinity. In his *De Trinitate*, St Augustine described the relationships between the persons of the Trinity as follows:

“Now love is of someone who loves and something is loved with love. So then there are three: the lover, the beloved, and the love (Bk 8 Ch 10). If then, any one of these three is to be specially called love, what more fitting than that this should be the Holy Spirit? In the sense, that is, that in that simple and highest nature, substance is not one thing and love another thing, but that substance itself is love and that love itself is substance whether in the Father or the Son or the Holy Spirit, yet that the Holy Spirit is especially to be called love” (Bk 15 Ch 17)

The threefold nature of proportionality is reinforced in traditional architecture. In order to gauge the degree of anything, the human mind needs a standard to measure it by. So in order to get a sense of the correspondence of a ratio to the divine order, it is helpful to have another to compare it with directly. Three terms are the minimum by which we can create two ratios that allow for the comparison of one with the other. When we see this in building we see three storeys of uneven size that communicate a sense of rhythmical progression, so that we sense a harmony in that the first storey relates to the second as the second relates to the third. The church of San Vitale in Ravenna is 5th century and the three layers in its design follow this format.



One quick demonstration of this is in St Giles, one of the main streets in the city of Oxford, where I lived until recently. On this street we have a range of styles ranging from the late gothic St John's College, built in 1550 through to the 20th century maths and engineering buildings (within sight of St Giles, up the Banbury Road). What is striking is the range of the styles (gothic, neo-classical, Victorian neo-gothic, Georgian) and sizes and how, despite this they all sit happily together. To my mind this is because they are built with reference to the traditional idea of harmonious proportion (in deference to tradition, if not Christian cosmography) and so are all in harmony with each other by virtue of the fact that they relate, by degrees at least, to the common invisible standard of the divine order. In some we see just three storeys, in others we see more, with each getting successively smaller; or with first greatest, the last smallest, but all between of

equal size, a multiplication of the mean of the proportion. Usually, the window size is a guide to these variations, but even if the windows do not vary, the sense of a greater size for a storey is communicated through the use of external features, such as sills or contrasting colour. For the most part, the apprehension of this a harmonious is intuitive, even if the design of it is mathematically reasoned. What is striking is how the modern buildings clash with all other styles on the street. Several photographs of the buildings of Oxford are shown at the end of this article.

It is noticeable to me that traditional designs also sit easily in a natural environment, adding to and perfecting a rural landscape. Modern buildings clash and fight against their surroundings.

Artists use a similar device, created intuitively, to lend grace and rhythm to a composition. The progression of lines that define form will be made to conform to a format whereby a sense of proportion is created. I was first taught to use these when painting, for example, the lines that are used to define the folds of cloth in the robes of a saint. If you pick any well painted icon and look, say at the flexed elbow, you will see these rhythmical lines defining the bunching of cloth. You would employ this in the painting to give it grace, even if it is not what you are seeing in nature in your model. Later, when I was student in Florence learning the Western academic method, I was drawing a cast of a sculpture of the Italian baroque sculptor, Bernini. This is a very different style from the iconographic but despite that I noticed that Bernini had these rhythmical lines tracing a lattice-work across the surface. In his sculptures of drapery, they are cut much more deeply than they would be in nature, so that this rhythmical effect is enhanced further. I exaggerated them in my drawing in accordance with what I had been taught by my icon painting teacher. When the head of the atelier came to give his daily critique, he looked at mine and said: 'You have natural baroque rhythm.' I didn't have the heart (or perhaps cheek) to tell him that it was as much 'iconographic rhythm'.



The Bible

The last article I wrote for the *New Liturgical Movement* discussed the hierarchy of numbers as discerned from the bible. Harmonious combinations of numbers were discerned from the bible too, for example when God directed the building or creation of something, He specified the dimensions. The Church Fathers analysed the degree to which the biblical proportionality reinforced those relationships derived by observation of the natural world, especially those of the pre-Christian Greeks.

Right up to the High Renaissance, these dimensions were viewed as an important source of proportion for architects. We have mentioned Palladio. Otto von Simson²⁰ describes a document by

²⁰ Otto von Simson, *The Gothic Cathedral*, p227, Harper and Row, 1964

another, a 16th century French architect, Philipert Dolorme. As well as the geometric and numerical principles already referred to, he asserts that the dignity of the architect's profession is proved by the fact that God himself has instructed him, in the biblical models, about the measures and proportions to employ. He proposes that architects study the 'sacred proportions' God gave to Noah, Moses, Solomon and Ezekiel.²¹ These are the dimensions of Noah's Ark, the Ark of the Covenant and the tabernacle of Moses, Solomon's temple, the celestial temple of the vision of Ezekiel. Another example is the heavenly city in the vision of St John in the Book of Revelation²².

Some of these biblical passages, especially Ezekiel, are very complex and to someone who is not an experienced biblical scholar (such as myself!), at times confusing. I do not pretend to have a deep understanding. I present them in order to draw attention to them as a traditional source.

We will focus here on the simpler relationships, as these will be the ones that are more easily applied in human activities, for example, compositional design in art and architecture. Whilst all of these biblical sources are referred to by architects, such as Palladio and Dolorme, the plans of buildings tend to reinforce only those ratios and proportions that correspond to those derived from other sources; for example with the arithmetical, geometric or musically derived harmonious proportions already discussed.

Noah's Ark

The length of the ark shall be 300 cubits, the breadth of it 50 cubits and the height 30 cubits
St Augustine linked this to the perfect human form of Christ by the observation that the ratios 1:6, 1:10 and 6:10 govern this structure.

The Ark of the Covenant

Two cubits and a half shall be the length thereof, one cubit and a half in breadth thereof and one cubit and a half in height thereof

This therefore contains the ratios of 6 : 10 and 1 : 1 and participates in both the instrumental musical and human musical proportions.

The Tabernacle of Moses

And thou shall make an altar of shittim wood, five cubits long, and five cubits broad; the altar shall be foursquare and the height thereof shall be three cubits.....The length of the court shall be an hundred cubits, and the breadth fifty cubits and the height five cubits.

This gives ratios of 1:1 and 5:3 in the altar. This last corresponds to the musical interval of a sixth which was used by Villard de Honnecort in the design of his church for the Cistercians. In

²¹ Genesis 6:15 (Noah's Ark); Exodus 25:10 (Ark of Covenant); Exodus 26, 27 (Moses' tabernacle); 1 Kings 6 (Solomon's temple); Ezekiel 40-44

²² Rev 21:16-17

addition to this there are detailed descriptions of boards that line the walls and curtains that hang over them that have unusual dimensions, for example, 1:20. Further discernment is need before I feel able to comment on these.

Solomon's temple

And the house which King Solomon built for the Lord, the length thereof was threescore cubits, and the breadth thereof twenty cubits, and the height thereof thirty cubits. And the porch before the temple of the house, twenty cubits was the length thereof, according to the breadth of the house; and ten cubits was the breadth thereof before the house...And against the wall of the house he built chambers round about, both of the temple and of the oracle: and he made chambers round about: the nethermost chamber was five cubits broad, and the middle was six cubits broad and the third was seven cubits broad...And then he built chambers against all the house five cubits high...And he built twenty cubits on the side of the house, both the floor and the walls with boards of cedar...And the house, that is the temple before it, was forty cubits long...And the oracle he prepared in the house within, to set there the ark of the covenant of the Lord. And the oracle in the forepart was twenty cubits in length, and twenty cubits in breadth, and twenty cubits in height thereof.

Although he does not discuss the dimensions, St Augustine, in his discourse on Psalm 126, sees the temple of Solomon as a prototype for the Lord's body just as with Noah's ark, but here as the mystical body of Christ, the Church. The ratios that run through this are 2:3, 1:2, 1:1 and 1:3. The chambers that skirt the wall of the 'house' get successively longer in an arithmetic progression: 5, 6, 7.

The Heavenly City of Ezekiel

This is described in Ezekiel Ch 40-44. The passage is too long to reproduce here. It is a very detailed and complicated description and its representation would require much study (and imagination), as such it is beyond the scope of this article²³. Unlike some of the other passages, for example Solomon's temple, this is not a set of instruction for earthly men to follow and build, but more a description of a vision of a future glorious city. Nevertheless, it is possible quickly to pull out some details that appear to correspond to the traditional proportionality of the Christian tradition as obtained from other sources. For example:

- 6:10:16 (Ez 40:48);
- 1:1 (eg Ez 41:1; 41:4; 42:19; 43:16)
- 2:1 (eg Ez 41:2) and 3:2 (Ez 41:4; 41:22);
- 1:2:4 a geometric proportionality (Ez 43:13).

There are also dimensions that I have not seen anywhere else (14:1:18; 5:2) and so would be reluctant to ascribe significance to them.

²³ Studies have been made in the past and one, for example, that survives is contained within a copy of the complete works of Richard of St Victor, published in 1518. Richard was a native of Scotland who became prior of the abbey of St Victor near Paris. He died in 1173. This has detailed diagrams of the heavenly city as imagined by Richard.

The Heavenly City in the Book of Revelation

And the city lieth foresquare, and the length is as large as the breadth: and he measured the city with the reed, twelve thousand furlongs. The length, the breadth and the height of it are equal. And he measured the wall thereof, an hundred and forty and four cubits, according to the measure of the man, that is, of the angel.

This passage uses ratios of 1:1 again. There is uncertainty as to exactly what the units referred to constitute. Some translations of units are not precise. So the Greek *stadion* is translated in the King James Bible as a furlong, for example, which is not precisely equivalent. As proportionality is a relative measure, the reliability of proportions taken from the bible is higher when the unit quoted is the same. For example, here the heavenly city of the Book of Revelation is quoted in the King James Bible as 144,000 furlongs square, so the ratio 1:1 can be inferred here. The walls on the other hand are 144 ‘cubits’ high. One can see that there is a numerical link between the height of the walls and their length, but in order to relate them directly one would need a deeper knowledge than I possess about the units used in the original text.

Table 1. The 10 proportional relationships of Boethius

Boethian name		Mathematical expression	Example
Arithmetic	Major	$a = b$	1 : 2 : 3 : etc... ad infinitum
Geometric	Major	$\frac{y}{x} = \frac{z}{y} = \frac{b}{a}$	1 : 2 : 4 :
Harmonic	Major	$\frac{z}{x} = \frac{b}{a}$	3 : 4 : 6 :
Fourth or Contrary to Harmonic	Minor	$\frac{z}{x} = \frac{a}{b}$	3 : 5 : 6 :
Fifth or Contrary to Geometric	Minor	$\frac{y}{x} = \frac{a}{b}$	2 : 4 : 5 : ...
Sixth or Contrary to Geometric	Minor	$\frac{z}{y} = \frac{a}{b}$	1 : 4 : 6 :
First of Four	Major	$\frac{z}{x} = \frac{a + b}{a}$	6 : 8 : 9
Second of Four	Minor	$\frac{z}{x} = \frac{a + b}{b}$	6 : 7 : 9
Third of Four	Major	$\frac{y}{x} = \frac{a + b}{a}$	4 : 6 : 7 ...
Fourth of Four	Minor	$\frac{y}{x} = \frac{a + b}{b}$	3 : 5 : 8 (or 6 : 10 : 16)

Table 2: The main musical harmonies expressed as ratios

Name from Boethius	Modern name	Ratio
Double	Octave	1:2 (or 6:12)
Sequitertian	Perfect fourth	3:4 (or 9:12)
Sesquialter	Perfect fifth	2:3 (or 8:12)
Triple	Octave plus fifth	1:3
Quadruple	Double Octave	1:4
(Not in Boethius)	Third	4:5
Sesquioctave	Tone	8:9
(Not in Boethius)	Sixth	3:5

Architectural examples



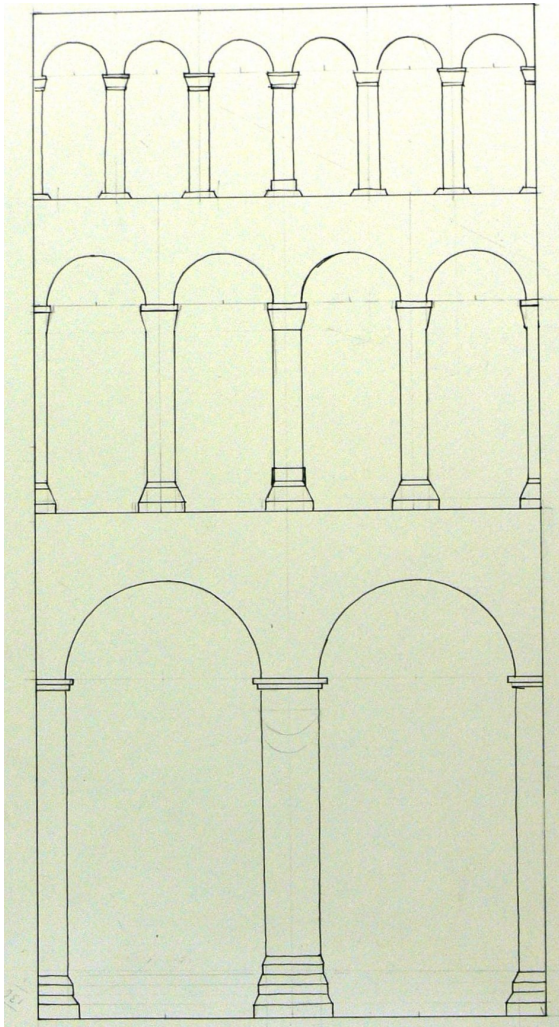
Left: Attingham House, Shropshire, England.
The design shows a 3, 4, 5 arithmetic progression in the layering of the storeys.

Below: three different views of St Giles in Oxford, showing the variety of styles, shapes and sizes of building happily sitting together.



Below left, middle: modern buildings, just off St Giles, showing rigidly even storey size, left, or varied storey size that does not correspond to traditional principles. Below, right: St John's College, Oxford, completed in 1550





A diagram (after Von Simson) and photograph of the Galleries of Angels, Abbey of St Michael, Hildesheim showing the 1:2:3 design in the number of arches





Modern icon of St Joseph, a Catholic devotion, left, and St Paul by the medieval Russian Andre Rublev, below, showing the graceful application of line employing proportional relationships



Left: example of calligraphic lines in the abstract that have a proportional relationship

